Risk Reduction Distributions Relating to the Benefits of Safety Efforts

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Validity, Quantitative Risk Analysis

Abstract

The benefit of including safety in new efforts has historically been described qualitatively. The objective of this paper is to update our recently defined equations showing value added from a safety activity and establish a method for the future quantification of a safety benefit. This paper approximates the distributions of mishap risk and compares the likely differences of mishap risk distributions before and after a safety effort. The result is a measure of safety improvement to a given effort. With this new approach, we show that, in many cases, the distribution can be approximated as lognormal with a small offset term.

Introduction

This paper computes mathematically modeled risk, i.e., severity multiplied by probability, for general mishaps before and after a system safety program is implemented. Such an attempt may be difficult for all such programs (ref. 1) given the complexity of risk and its elements as defined in MIL-STD 882E (ref. 2). In our earlier work (refs. 3-6), the distribution of mishap probability was characterized given a combination of practical experience, simulation based testing and incorporating expert opinion. Here, mishap probability and consequence data before and after the safety program is used to compute the initial risk distribution and estimate the financial risk reduction distribution that might be generated by a safety program.

In our paper at this conference last year (ref. 6), we showed that any viable safety program would not only reduce the risk, but also the relative uncertainty in that risk. This is due to the variability of the forces in the plane of mishap probability and consequence. In order to illustrate this effect, we approximated the distributions of both the probability and consequence, resulting in a risk, which is the product of the probability and the consequence. In order to model the savings created by a safety program, we expanded our definition of the lognormal distribution to allow an offset. This is still an approximation, but we were able to describe the cumulative distribution for the difference. We discuss the validity of the parameters as derived from the raw mishap distributions and demonstrate that the mean and variance can be derived by simple formulae. Limitations of the formulae are reviewed, and applicability conditions are discussed.

Analysis

First, we consider the risk characterization with a lognormal approximation and derive the associated parameters from our earlier models (refs. 3-6). The distribution is then generalized to include an offset, which is the minimum risk difference allowed. Positive results indicate a risk reduction, but it is also remotely possible, but unlikely, to increase the risk. Since both risk distributions are defined from zero to infinity, the risk difference is defined from –∞ to +∞, however, in practice, the risk difference density is often very small for differences less than small negative numbers.
Figure 1 shows the computed probability distributions for risk obtained from simulated data in last year’s study (ref. 6). The abscissa gives the risk expressed in dollars per 1000 miles driven. The distributions before and after the (hypothetical) safety program are drawn in blue and red, respectively. These risk distribution curves formed the basis for the calculation of the risk reduction probability distribution. The first intersection is the result of the functional characterization that was used for the risk. The second intersection is very important as it points out where the benefit of the safety program presents itself through a reduced probability of high risk. Note the higher probability of low risk and lower probability of high risk will always be the case for any rational safety program.

A general parameter, $X$, with a lognormal distribution can be written as

$$X = \delta + \exp(\mu + \sigma Z)$$

(1),

where $Z$ is a standard Normally distributed parameter,

- $\delta$ is an offset parameter (usually negative, in our case),
- $\mu$ is the median parameter (median = $\delta + \exp(\mu)$), and
- $\sigma$ is the shape parameter (standard deviation of ln($X - \delta$)).

In last year’s paper (ref. 6), we assumed that $\delta$ was zero as this is usually valid for risk characterization. However, for describing risk reduction, a small negative value for this parameter is often required. Hence, we have expanded the lognormal definition to include a non-zero offset. We do not usually solve for $\delta$ in a formal sense, as we might have to compute the logarithms of negative numbers, which, of course, are complex numbers. Therefore, we generally select a small value that gives a superior fit for the higher percentile data.

The mean, $m$, and variance, $V$, of $X$ in terms of $\mu$, $\sigma$ and $\delta$ are:

$$m = \delta + \exp(\mu + \sigma^2/2)$$  \hspace{1cm} (2), and

$$V = [\exp(\sigma^2) - 1]\exp(2\mu + \sigma^2)$$  \hspace{1cm} (3).
For any general distribution with a minimum abscissa of δ, mean value, m, and variance, V, we can obtain a corresponding lognormal distribution approximation by solving equations (2) and (3) for μ and σ² as follows:

\[
\mu = \ln\left(\frac{m-\delta}{\sqrt{1+V/(m-\delta)^2}}\right) \quad (4) \\
\sigma^2 = \ln(1+V/(m-\delta)^2) \quad (5).
\]

We assume that the mean and variance of the risk can be estimated before and after the safety program, and then compute parameters for the savings generated from the parameters before and after the safety program as given in the following table:

**Table 1 - Computation of Savings Statistics**

<table>
<thead>
<tr>
<th></th>
<th>Before</th>
<th>After</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>m_B</td>
<td>m_A</td>
<td>m_S = m_B - m_A</td>
</tr>
<tr>
<td>Variance</td>
<td>V_B</td>
<td>V_A</td>
<td>V_S = V_B + V_A</td>
</tr>
<tr>
<td>Offset parameter</td>
<td>0</td>
<td>0</td>
<td>Small negative number obtained by trial and error</td>
</tr>
</tbody>
</table>

Figure 2 illustrates the numerical process we used for computing the risk reduction distribution. The savings amounts for the before and after cases were estimated from the lognormal parameters for all integer cumulative percentiles as well as 0.5 percent and 99.5 percentiles. This created two lists, each containing 101 percentiles (one through 99 in additions to 0.5 and 99.5). We then computed the difference between every member of the list before the safety program and every member on the list after the program. This yielded a total of 10,201 differences, whose statistics were computed. The savings distribution parameters were modeled as a lognormal population with a negative offset parameter.

![Computation of Risk Reduction Diagram](image)
Discussion of Results

With our new approach this year, we used the distributions shown in Figure 1 where, we simulated 50 random realizations for each case. The computed parameters before and after the safety program are specified in Table 2 together with those from the savings. Note that while the mean and variance of the savings can be estimated from the formulae given in Table 1, the median and shape parameters must be obtained in a more complicated fashion. Equations (2) and (3) were used to calculate the mean and variance for the before and after cases, while Equations (4) and (5) were used to compute $\mu$ and $\sigma$ for the savings.

Table - 2 Results for Accident Repair Cost Parameters

<table>
<thead>
<tr>
<th></th>
<th>Before</th>
<th>After</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ($ per 1000 miles)</td>
<td>210.5</td>
<td>28.1</td>
<td>182.4</td>
</tr>
<tr>
<td>Variance ($ per 1000 miles)$^2</td>
<td>270242</td>
<td>500</td>
<td>270742</td>
</tr>
<tr>
<td>Offset ($ per 1000 miles)</td>
<td>0</td>
<td>0</td>
<td>-50</td>
</tr>
<tr>
<td>Median parameter, $\mu$</td>
<td>4.369</td>
<td>3.091</td>
<td>4.100</td>
</tr>
<tr>
<td>Shape parameter, $\sigma$</td>
<td>1.4</td>
<td>0.7</td>
<td>1.487</td>
</tr>
<tr>
<td>Median ($ per 1000 miles)</td>
<td>79</td>
<td>22</td>
<td>45</td>
</tr>
</tbody>
</table>

The cumulative savings distribution is given in Figure 3, together with the computed lognormal fit. The value of the offset was -$50 per 1000 miles driven, and the parameters we obtained from Equations (4) and (5) are specified on the graph. Note that the savings can be negative, but in practice, the probability of any value below $\delta$ (-$50 per 1000 miles driven) is very small.

Figure 3 - Cumulative Distribution of Savings Resulting from Safety Program

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Now it is possible to judge the value of the expenditures on risk reduction activity and generate percentile confidence that the safety program will be justified. Even a very high risk reduction program cost could easily provide a significant return on investment. In this case there is data that could reasonably be expected to be collected for every accident, so that a fitting process worked well. In practice, the parameters might be more difficult to obtain, but the lognormal approximation is very often a good starting point. It should be noted that all of the equations used in this approach are also applicable to the performance improvement domain with the simple expediency of interpreting the risk as that of system performance failure.

The lognormal assumption for risk is an approximation whose validity can be assessed for real-world situations. The mishap probability has a Beta distribution while the mishap consequence distribution is highly variable. Nevertheless, our approximation here appears to be conservative (ref. 6), and can be used with reasonable consequence for many cases.

Conclusions

This result allows management to estimate the percentile probability of returns on investment for any planned safety program. The lognormal assumption is an approximation that must be checked, but is generally very helpful. The conclusions for the safety risk also apply to performance risk because the safety and performance risk planes have identical characterizations.

References


Biographies

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