Reliability Enhancement Test Evaluation Based on Accelerated Growth Model

Yuankai Gao, MS; Beihang University; Beijing, Beijing, China

Xiaogang Li; Ph.D.; Beihang University; Beijing, Beijing, China

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Abstract

Reliability enhancement test has been widely used as a practical technology to improve product reliability. Because the purpose of the RET is to make the product more robust in a short time, it can only qualitatively evaluate the product reliability and cannot quantitatively obtain the product reliability index. Therefore, in order to improve product robustness in a short period of time, it is also possible to quantitatively calculate the reliability level of the product, this paper introduces an evaluation method for accelerating the reliability growth test. First of all, the acceleration model (Arrhenius model) in the accelerated life test is used to convert the test time at each stage of high temperature step stress into the corresponding life time under normal stress. In the process of reliability enhancement test, continuous improvement of the product is accompanied by an increase in the inherent reliability of the product. Therefore, we analyze the data under normal stress through the reliability growth model (Duane model). Then calculate the life of the product at the end of the test to achieve a quantitative assessment of the reliability enhancement test. On this basis, a variable acceleration model with time varying acceleration is also derived. Finally, the rationality and availability of the evaluation method are further verified by enumerating concrete calculation examples.

Introduction

The development of reliability test technology has formed two major types of methods, stimulation test and simulation test, since its birth. The simulation test is to simulate the actual use conditions of the mission profile as much as possible, plus the design margin to ensure the reliability of the product. A typical representative is the reliability growth test. However, such tests for high reliability and long-life products are not suitable, because this will not only lead to a longer development cycle, but will also greatly increase the cost of the test. Therefore, the stimulation test has emerged. The stimulation test does not simulate the real conditions of the equipment mission profile. Instead, the method of artificially applying accelerated stress accelerates the stimulating of potential defects, and improves the product reliability through analysis and improvement to achieve product robustness. Typical representative is reliability enhancement test. However, the reliability enhancement test cannot be used to quantitatively evaluate the reliability index of the product at the end of the test as the reliability growth test, but can only qualitatively evaluate the product reliability. Therefore, how to quantitatively evaluate the reliability of the tested product by the failure data of RET becomes a hot topic in the field of reliability testing technology. Firstly, the accelerated time model in the accelerated life test (Arrhenius model) was used to extrapolate the test time under the high temperature step stress to the corresponding normal stress life time. Secondly, considering the continuous improvement of the product in RET, the inherent reliability of the product is increasing
continuously, so the extrapolated data is analyzed by the reliability growth model (Duane model). As a result, a quantitative assessment of reliability enhancement testing is achieved. On this basis, a variable acceleration model with time varying acceleration is also derived. Finally, the specific examples were used to further verify the usability and correctness of the method.

1 Basic Theory

1.1 Accelerated Model

Accelerated models can be divided into three categories according to their proposed methods: physical acceleration model, empirical acceleration model, and statistical acceleration model. Several phenomena often occur during reliability enhancement testing, including mechanical fatigue damage, wear, electron migration, and chemical reactions. These have established different mathematical models and respond differently to different incentives or actions. Arrhenius model is mainly used to describe the failure mechanism of non-mechanical, chemical reaction, corrosion, material diffusion or migration process. For products whose life is subject to an exponential distribution. According to the Arrhenius model, the expression of a product's life-time characteristics with temperature stress can be obtained, ie:

\[ L = A \times e^{\frac{E_A}{kT}} \]  

Where A is a constant, \( E_A \) is activation energy, k is Boltzmann's constant, and T is absolute temperature.

\[ \frac{L_A}{L_N} = \frac{A \times e^{\frac{E_A}{kT_A}}}{A \times e^{\frac{E_A}{kT_N}}} = e^{\frac{E_A}{k} \left( \frac{1}{T_A} - \frac{1}{T_N} \right)} \]  

In the formula, \( L_A \) is the life characteristics of the product under accelerated stress levels; \( L_N \) is the product life characteristics under normal stress levels; \( T_N \) is the normal level of temperature stress; \( T_A \) is the temperature stress at the acceleration level. Further deductions are:
1.2 Reliability Growth Model

Reliability growth tests must have a growth model. The reliability growth model can track the current reliability level of the product in the test process based on the number of failures or failure time series provided during the reliability growth test, and quantitatively evaluate the reliability level that the product ultimately achieves after the test is completed. The reliability growth model is a mathematical expression that describes the law and general trend of product reliability growth during the reliability growth test. Reliability model is divided into continuous type and discrete type. The commonly used models are Duane model, AMSAA model and so on. The Duane model is first introduced. The method used in this article is the Duane model, so only the Duane model is introduced here. The Duane model expression represented by the instantaneous MTBF is:

$$MTBF(t) = \frac{t^m}{a(1-m)}$$

(4)

The Duane model expression expressed as cumulative MTBF is:

$$MTBF(t) = \frac{t^m}{a}$$

(5)

Where, \(m\) is the growth rate and \(a\) is a constant.

2 Quantitative Assessment of RET

2.1 Data Extrapolation Methods

Accelerated model analysis and processing of accelerated test data and extrapolation of accelerated test data to failure data under normal stress conditions are the core issues in the study of reliability assessment methods for high acceleration tests. During the test, as the test stress level continues to increase, product failures continue to be revealed, it is necessary to analyze the failure mechanism of the product and then improve the product to achieve an increase in the inherent reliability of the product. This process makes the activation energy needed to activate the fault increase as the inherent reliability of the product increases. From the perspective of activation energy, the inherent reliability of the product will increase stepwise with each improvement of the product, and the activation energy will also increase stepwise (as shown in Figure 1), at each stress level. The failures have different activation energy and different acceleration factors.

Obviously, the activation energy in the time zone \((t_i, t_{i+1})\) of the i-th failure to the \((i+1)\)-th failure in the RET test is equal to the activation energy in the time interval \((t_{i+1}, t_{i+1})\) extrapolated to the normal stress level. We need to record the product's continuous (failure) time at different stress levels. The failure time of the test unit at each stress level (in terms of
cycle times or time) must be converted in stages to the corresponding normal stress. This can be achieved by experimenting with "deceleration" (ie: extrapolation of data) and converting the number of trials to actual duration. We denote the activation energy at the i-th failure at the j-th acceleration stress level as $E_{a(ij)}$, and the acceleration factor is recorded as:

$$A_{F_j} = e^{\frac{E_{a(ij)} + 1}{k(T_j - T_h)}}$$

(6)

For ease of understanding, the high-temperature step stress test was used as an example to illustrate, so the Arrhenius model was selected. When the failure data of the product at the accelerated stress level is recorded (only one failure occurs at each stress level), the acceleration factor can be extrapolated to the i-th failure data of the product at the normal stress level. Therefore equation 6 and equation 3 can be used to further derive:

$$L_{N_l} = L_{A_{F_j}} = L_{A_{i_j}} \times e^{\frac{E_{a(ij)} + 1}{k(T_j - T_h)}}$$

(7)

In the formula, $L_{A_{F_j}} = t_j - t_{(j-1)}$ is the time from the (j-1)-th failure to the j-th failure at the i-th stress level; $L_{N_l} = t_{(j+1)} - t_{(j+1)}$ is the time from the j-1-th failure to the j-th failure at extrapolation to the normal stress level.

In the above case, for ease of understanding we assume that only one failure occurs in each stage of the product, and the feasibility of data extrapolation is studied from the perspective of activation energy. However, the number of failures in each phase of the actual test process is often more than one, so the following describes how the data is extrapolated when the number of failures in each phase is not less than one. The specific process is as follows:

It is assumed that the product undergoes reliability enhancement tests at different levels of environmental stress in time interval $\{0, T\}$. The number of stress levels experienced in the test was m ($m \geq 2$), the number of failures occurred was r ($r \geq n-1 > 1$), and the number of improvement was n-1. This is an n-stage strengthening test under different levels of environmental stress, from the beginning of the test to the first improvement to the first phase, with the adjacent two improvements being a phase, and the n-1th improvement to the end of the test as n stage. If we consider from the perspective of failure rate, the failure rate of the i-th stage product under the reference stress is $\lambda_i$ (i=1,...,n). Generally, each improvement reduces the failure rate, ie, $\lambda_1 > \lambda_2 > ... > \lambda_n$. The details of the test are as follows:

The product started the test from time $t_{1,0} = 0$. In the first stage, the stress level applied on time interval $(t_{1,j-1}, t_{1,j})$ is $f_{1,j}$ ($j = 1,...,k_1$), and $r_1$ ($r_1 \geq 1$) failures occur in total. The stress at the time of the $l$-th failure was recorded as $f_{1,l}$ ($l = 1, ..., l$). The first $r_l-1$ failures are only repaired, and the $r_l$-th failure occurs at the $t_{1,k_1}$, after which improvements are made. The corresponding baseline test time for the first phase is:
According to the above idea, in the $i$-th (i=1, 2, ..., n-1) stage, the stress level applied by the equipment at time interval $(t_{i,j-1}, t_{i,j})$ is $f_{i,j}$ ($j=1,...,k_i$). A total of $r_i$ ($r_i \geq 1$) failures occurred during the $(t_{i,0}, t_{i,k_i}]$ period, and stress $f*_{i,l}$ ($l=1, \ldots, r_i$) at the time of the $l$-th failure. The previous failures were only repaired, and the $l$-th failure occurred at the $t_{i,j}$, after which improvements are made. The corresponding baseline test time for the $i$-th phase is:

$$T_i(A) = \sum_{j=1}^{k_i} (t_{i,j} - t_{i,j-1}) e^{\frac{A - A_i}{f_{i,j}}}$$

(8)

In the $n$-th stage, the stress level applied by the device during the $(t_{n,j-1}, t_{n,j})$ is $f_{n,j}$ ($j=1,\ldots,k_n$), and $r_n$ ($r_n \geq 1$) failures occur in $(t_{n,0}, t_{n,k_n}]$. Failures were only repaired. The stress at the moment of $l$-th failure $f*_{n,l}$ ($l=1, \ldots, r_n$). The test ended at time $t_{n,k_n} = T$. Therefore, the corresponding baseline test time is:

$$T_n(A) = \sum_{j=1}^{k_n} (t_{n,j} - t_{n,j-1}) e^{\frac{A - A_i}{f_{n,j}}}$$

(9)

From the above, the corresponding likelihood functions for failure probability density function are:

$$L(A; \lambda_1, \ldots, \lambda_n) = \prod_{i=1}^{n} \prod_{l=1}^{r_i} \lambda_i e^{\frac{A - A_i}{f_{i,l}}} e^{-\lambda_i T_i(A)}$$

(11)

The log likelihood function is further derived as:

$$l(A; \lambda_1, \ldots, \lambda_n) = \sum_{i=1}^{n} [r_i \ln \lambda_i + \sum_{l=1}^{r_i} (\frac{A - A_i}{f_{i,l}}) - \lambda_i T_i(A)]$$

(12)

Likelihood equations are:

$$\frac{\partial l}{\partial \lambda_i} = \frac{r_i}{\lambda_i} - T_i(A), i = 1,\ldots,n,$$

(13)

$$\frac{\partial l}{\partial A} = \sum_{i=1}^{n} \left[ \sum_{l=1}^{r_i} (\frac{1}{f_{i,l}} - \frac{1}{f_{i,l}}) - \lambda_i \sum_{j=1}^{k_i} (t_{i,j} - t_{i,j-1}) \exp\left(\frac{A - A_i}{f_{i,j}}\right) (\frac{1}{f_{i,j}} - \frac{1}{f_{i,j}}) \right]$$

(14)

If equation 13 is equal to zero then there are:
\[ \lambda_i = \frac{r_i}{T_i(A)}, i = 1, \ldots, n, \]  \hspace{1cm} (15)

Bringing equation 15 into equation 14 has:

\[
h(A) = \frac{\frac{\partial l}{\partial A}}{\sum_{i=1}^{n} \frac{1}{T_i(A)} - \frac{1}{f_{i,j}^*}} - \frac{r_j \sum_{j=1}^{k_i} (t_{ij} - t_{ij-1}) \exp\left[\frac{A}{f_{ij}^*} \left(\frac{1}{f_{ij}^*} - \frac{1}{f_{ij}}\right)\right]}{\sum_{j=1}^{k_i} (t_{ij} - t_{ij-1}) e^{\frac{A}{f_{ij}}}} \]  \hspace{1cm} (16)

Because \( h'(A) < 0 \), \( h(A) \) are strictly monotonic decreasing, the unique solution of \( h(A) = 0 \) exists, and is the maximum point of the log-likelihood function. (Proved to skip)

When \( A \to 0^+ \),

\[
\lim_{A \to 0^+} h(A) = \sum_{i=1}^{n} \left( \sum_{j=1}^{k_i} \left( -\frac{1}{f_{i,j}^*} \right) - r_j \sum_{j=1}^{k_i} (t_{ij} - t_{ij-1}) \left( -\frac{1}{f_{i,j}^*} \right) \right) > 0 
\]  \hspace{1cm} (17)

When \( A \to \infty \),

\[
\lim_{A \to \infty} h(A) = \sum_{i=1}^{n} \left( \sum_{j=1}^{k_i} \left( -\frac{1}{f_{i,j}^*} \right) - r_j \left( -\frac{1}{\max_{i \leq j \leq k_i} f_{i,j}} \right) \right) \leq 0 
\]  \hspace{1cm} (18)

Therefore, the necessary and sufficient condition for the existence and unique solution of the solution of the likelihood equation is:

\[
\sum_{i=1}^{n} \left( \sum_{j=1}^{k_i} \left( -\frac{1}{f_{i,j}^*} \right) - r_j \left( -\frac{1}{\max_{i \leq j \leq k_i} f_{i,j}} \right) \right) < 0 
\]  \hspace{1cm} (19)

And,

\[
h(0) = \frac{\partial l}{\partial A} = \sum_{i=1}^{n} \left( \sum_{j=1}^{k_i} \left( -\frac{1}{f_{i,j}^*} \right) - r_j \sum_{j=1}^{k_i} (t_{ij} - t_{ij-1}) \left( -\frac{1}{f_{i,j}^*} \right) \right) > 0 
\]  \hspace{1cm} (20)

Note that \( f_{i,j}^* \) is the stress applied at the \( l \)-th failure in the \( i \)-th stage, and \( \max_{i \leq j \leq k_i} f_{i,j} \) is the maximum stress applied at the \( i \)-th stage. The right end of equation 19 should be less than or equal to 0. Therefore, as long as not all failures occur at the highest stress level of their stage, equation 19 must be true. If A's maximum likelihood estimate \( \hat{A} \) exists, \( T_i(\hat{A}) = \sum_{j=1}^{k_i} (t_{ij} - t_{ij-1}) e^{\frac{A}{f_{ij}}} \) can be used instead of \( T_i(A) \). Therefore, the reliability growth process under the above-mentioned different levels of high accelerating environmental stress can be transformed into the reliability growth problem under the same environment, so that data analysis methods can be used for analysis.
In summary, considering that there are few samples of accelerated test products at the engineering development stage, the above method is aimed at the accelerated test of single products. In fact, in the search for the stress model parameter A, as much as possible, the test information of all exponential products with the same working environment and similar test conditions should be used. Because in such cases, these products should have the same environmental factors, and the obtained parameter values are more accurate.

2.2 Extrapolation Data Processing

In the process of implementing reliability enhancement tests, the weak points of the product are continuously exposed as the test stress level increases. This is actually a test-improvement-retest process. Therefore, as the stress level increases, it is necessary to continuously improve the products for the design defects exposed in the tests so as to achieve a rapid increase in the inherent reliability of the products. This is actually a test-improvement-retest process. Traditional reliability growth tests must also be tested-improved-retested until they meet the requirements. From the above, it can be seen that the growth process of the reliability growth test has something in common with the growth process of the reliability enhancement test. For example, product reliability growth is usually only related to reducing the impact of systemic weaknesses; The failure of systemic weak points was revealed and failure analysis was carried out, improvement measures were taken, and the effectiveness of the improvement measures was verified; the reliability of the product was continuously improved during the test. The traditional reliability growth test after the end of the test, according to the failure data of each growth phase to select the appropriate growth model, and then the product can be quantitatively evaluated at the end of the test reliability. Therefore, we can consider the extrapolated accelerated test data as a set of reliability growth test data to quantitatively evaluate the reliability of the product at the end of the accelerated test through the reliability growth test evaluation method.

The indispensable information in the data analysis of the reliability growth test is the failure time. When calculating the failure time under normal stress by the failure data in the accelerated test, extrapolation of the test data is required through the data extrapolation method in the 2.1, and the extrapolated data obtained are arranged in ascending order and analyzed in accordance with the reliability growth model. Due to the fact that the number of products under test and the type of stress during the test are different, the types and characteristics of the extrapolated data are different. When selecting a reliability growth model, the selection of the most appropriate growth model needs to be selected based on the type and characteristics of the extrapolated data. The reliability growth model is not only an important tool to describe the product reliability changes regularly with the test time, but also an effective method to evaluate the reliability status of the product at any moment of change.

2.3 Duane Model Evaluation Method

According to the results recorded during the experiment, the growth model was selected for evaluation. The commonly used models were the Duane model and the AMSAA model. The AMSAA/Crow model is usually preferred, and the Duane model is recommended only if a few experiments fail. Since the product is in the development stage during the accelerated testing phase, the number of samples is small, and the failure data after the RET test is less, so the
growth model selected for the convenience analysis is the Duane model.

As a time function model, Duane model can dynamically evaluate the reliability of products and can perform reliability prediction. When the Duane model is used to estimate the product lifetime, the estimation of parameters in the model is mainly used, and the commonly used parameter estimation methods are divided into map estimation and least squares estimation.

2.3.1 graph estimation

To perform the graph estimation of the Duane model, follow the following steps to plot the growth curve based on the Duane model.

① As the test progresses, continuously record the cumulative number of failures N(t) of the product under test and the corresponding cumulative test time t; ② Calculate the corresponding t/N(t) for each selected time t value. If t is too large, transform the ratio d to it. ③ Mark each point (t, t/N(t)) on double logarithmic paper; ④ If the plotted points can form a straight line, then the Duane model can be used. On the plotted Duane curve, the following method is used to calculate the estimated value of each parameter.

1) The parameter m is given directly by the slope of the line;

2) The parameter a can be determined according to the ordinate value of the intersection point of the Duane curve and the vertical axis 1/N(1), ie:

\[ a = \begin{cases} (1/N(1))^{-1}, & \text{No transformation} \\ (1/N(1))^{-1} \cdot d^{-1}, & \text{Transformation} \end{cases} \]  

\[ (21) \]

3) MTBF value is \( \hat{\theta}(t) = \frac{t^m}{a(1-m)} \), and \( \hat{R} = \exp\left(-\frac{t}{\hat{\theta}(t)}\right) \).

2.3.2 least squares estimation

The accuracy of the least-squares estimation is higher than that of the map, and it can be used for both complete data analysis and packet data analysis. It can be used for both the failure-closure analysis and time-closure analysis. However, for incomplete data, it is required to have a certain amount of cumulative failures.

The Duane model uses double logarithmic coordinates and can be expressed as

\[ \ln \theta_{\epsilon}(t) = -\ln a + m \ln t \]  

\[ (22) \]

Accumulated MTBF is obtained from the cumulative test time \( t_1 < t_2 < ... < t_n \) and the corresponding cumulative failure times \( N(t_1), N(t_2), ..., N(t_n) \).

\[ \theta_{\epsilon}(t_j) = \frac{t_j}{N(t_j)} (j = 1, 2, ..., n) \]  

\[ (23) \]
According to the Duane model there are
\[
\ln \theta_j(t_j) = -\ln a + m \ln t_j + \epsilon_j \quad (j = 1, 2, ..., n)
\]  
(24)

Therefore, the least squares estimate for \( m \) and \( a \) is
\[
\hat{m} = \frac{n \sum_{j=1}^{n} \ln \theta_j(t_j) \ln t_j - (\sum_{j=1}^{n} \ln \theta_j(t_j)) \times (\sum_{j=1}^{n} \ln t_j)}{n \sum_{j=1}^{n} (\ln t_j)^2 - (\sum_{j=1}^{n} \ln t_j)^2}
\]
(25)

\[
\hat{a} = \exp \left\{ \frac{1}{n} \left[ m \sum_{j=1}^{n} \ln t_j - \sum_{j=1}^{n} \ln \theta_j(t_j) \right] \right\}
\]
(26)

So, the least square estimate of instantaneous failure rate \( \lambda(t) \) is
\[
\hat{\lambda}(t) = a(1 - \hat{m}) t^{-\hat{m}} \quad (0 < t \leq t_n)
\]
(27)

Since the repairable product does not improve or correct after time \( t_n \), it can reasonably be assumed that after the product is shaped, its failure time obeys an exponential distribution, ie, the least square estimate of MTBF at time \( t_n \) is
\[
\hat{\theta}(t) = \frac{t_n^{\hat{m}}}{a(1 - \hat{m})}
\]
(28)

After further shaping, if the task time of the product in use is \( t \), the least square estimate of its reliability is:
\[
\hat{R} = \exp \left( -\frac{t}{\hat{\theta}(t)} \right)
\]
(29)

After the parameter estimation, the Duane model needs to be tested for goodness of fit to further verify the closeness of the test data to the fitted line. This article will not be described again.

2.4 The Establishment of Variable Acceleration Model

As shown in Figure 1 of the 2.1, as the reliability of the product improves constantly, the activation energy needed to stimulate the potential defects of the product will also increase with the increase of the inherent reliability of the product. In accordance with the idea of using the Duane model to extrapolate data in the 2.2, we can further extend the stepwise growth relationship that can change with time to a continuous growth relationship. The activation energy in the time zone \( t_i, t_{i+1} \) in the RET from the \( i \)-th failure to the \( i+1 \)-th failure is equal to the activation energy in the time interval \( t_{i, i+1} \) pushed to the normal stress level. For an exponential life-time product, the MTTF and MTBF of the product are the same. We can reasonably assume that the current MTBF of the product is equal to the current MTTF of the product. Therefore, the following relationship can be obtained from (1) and (4):
\[ MTBF = \frac{t_0^m}{a(1-m)} = \hat{MTBF} = A e^{-\frac{E_a(t)}{RT_0}} \]  

(30)

Where, \( t_0 \) is the cumulative test time of the product in the normal stress environment, \( \hat{MTBF} \) is the instantaneous \( MTBF \) of the product at time \( t_0 \), and \( \hat{MTTF} \) is the mean time to failure of the product at the time \( t_0 \). Further derivation,

\[ E_a(t_0) = kT_0 \times \ln \frac{t_0^m}{Aa(1-m)} \]  

(31)

From equation 31, we can regard the stepwise growth process of activation energy \( E_a \) as a continuous growth with cumulative test time under normal environmental stress. Therefore, at any moment \( t_1 \) under the accelerated stress environment, the product has the same activation energy as the moment \( t_0 \) in the normal stress environment. From the calculus point of view, the small intervals \( \Delta t_1 \to 0 \) and \( \Delta t_0 \to 0 \) are taken respectively after the \( t_1 \) and \( t_0 \) moments, and the activation energy of the product can be considered to be constant in the small interval. If the value is \( E_a(t_0) \), then \( \Delta t_1 \) and \( \Delta t_0 \) satisfy equation 7, which is

\[ \Delta t_0 = \Delta t_1 \times e^{\frac{E_a(t_0)}{k} \left( \frac{1}{T_0} - \frac{1}{T_1} \right)} \]  

(32)

By equation 32 we can get the integration relationship between \( t_1 \) and \( t_0 \) is

\[ t_1 = \frac{E_a(t_0)}{k} \left( \frac{1}{T_0} - \frac{1}{T_1} \right) dt_0 \]  

(33)

According to equation 31 and equation 33, the relationship between cumulative test time \( t_0 \) under normal stress level and cumulative test time \( t_1 \) under accelerated stress environment can be obtained as

\[ t_0 = \left[ t_1 \left( \frac{1}{T_1} - \frac{1}{T_0} \right) T_1 m + 1 \right] \times \left( Aa(1-m) \left[ \frac{1}{T_0} \frac{1}{T_1} \right] \right) \left[ \frac{1}{T_0} \frac{1}{T_1} \right]^{m+1} \]  

(34)

Substituting equation 34 into equation 31, the expression of the activation energy \( E_a \) with the cumulative test time for the reliability enhancement test can be expressed as

\[ E_a(t_1) = kT_0 \times \ln \frac{t_1 \left( \frac{1}{T_1} - \frac{1}{T_0} \right) T_1 m + 1 \times \left[ Aa(1-m) \left[ \frac{1}{T_0} \frac{1}{T_1} \right] \right] \left[ \frac{1}{T_0} \frac{1}{T_1} \right]^{m+1}}{Aa(1-m)} \]  

(35)

When the expression of the activation energy \( E_a \) is determined, the expression of the Arrhenius acceleration model that changes with the RET time is
Thus, the expression that the acceleration factor varies with the time of the reliability enhancement test is

$$L = Ae^{\frac{E_a(t_f)}{kT}}$$

(36)

The conditions for establishing the variable acceleration model mentioned above must meet the following three conditions:

1. At each stress level, the life of the product is subject to the exponential distribution, and after the product is improved, the distribution type of the product life is not changed and only the parameters of the distribution are changed.

2. The relationship between the product's characteristic lifetime and the accelerated stress level satisfies the Arrhenius acceleration model, and the data extrapolated to the normal stress level satisfies the Duane growth model.

3. The remaining life of the product is only related to the accumulated failure part and the current stress level, but has nothing to do with the accumulation mode.

The above assumptions for the product at the prototype stage can be satisfied. First of all, the distribution of life of most electronic and electromechanical products obeys the exponential distribution. Secondly, the improvement of product design defects is usually carried out locally, and will not change the basic properties of the product, that is, electromechanical products will still be electromechanical products. Therefore, the first condition is satisfied. The application of the Arrhenius accelerated model in accelerated life test assessment has validated its effectiveness. In the case where failures caused by tests conducted in different stress environments take the same degree of corrective measures, it can be assumed that the growth rates calculated from the extrapolated test data are equal. So, the second condition can also be satisfied. The third condition is that Nelson proposed based on failure physics, which is mainly for the problem of extrapolation of step-by-step test plan data. It can also be satisfied. Therefore, the variable acceleration model proposed in this paper can express the relationship between acceleration factors over time.

3 Case Study

In order to verify the correctness and feasibility of the method, this paper illustrates the reliability enhancement test of a certain avionics. The sensitive stress of the electronic product is temperature, so only the reliability enhancement test under temperature step stress is considered. After failure analysis, the failure mechanism at each stage is unchanged. In the 2.1, the data extrapolation method (9) represents that each test phase is divided according to the number of improvements to the product, and in each phase, according to the stress level of the product, it is divided into several sub-stages ,where each fault only repair does not improve(repair time is very short),and only taking improvement actions at the last failure of each phase. The information after the test is shown in the following Table 1.
### Table 1 — Test Data

<table>
<thead>
<tr>
<th>Experimental phase-i</th>
<th>Number of failures</th>
<th>Sub-phases-j</th>
<th>The last failure time of phase-i</th>
<th>Failure time under normal stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>1.3</td>
<td>128.05</td>
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<td>2</td>
<td>3</td>
<td>2</td>
<td>2.6</td>
<td>390.75</td>
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<td>3</td>
<td>2</td>
<td>4.3</td>
<td>665.43</td>
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<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>5.8</td>
<td>1149.56</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>8.1</td>
<td>1833.03</td>
</tr>
</tbody>
</table>

![Duane curve of product test data](image)

**Figure 2**— Duane curve of product test data

The Duane curve of the reliability enhancement test can be obtained after the parameters of Table 1 are estimated by the least-square method.

Therefore, according to the estimation method in 2.2.2, the growth rate, the MTBF at the end of the test, and the reliability at time t can be calculated. The specific results are as follows:

1. Growth rate \( m = 0.6638 \), \( a = 0.7035 \); MTBF at the end of the test = 619.8h
2. Product specified life time 100 h, so reliability at \( t = 100 \) h: \( R(t) = 0.85 \).

### 4 Conclusion

In this paper, by using the method of life assessment in accelerating reliability growth test, we propose a method for quantitatively evaluating product life based on the comprehensive use of accelerated test data (Arrhenius model) and reliability growth model (Duane model). The extrapolation formulae for the experimental data under accelerated stress and the test data under normal stress were deduced, and the growth process of extrapolated test data was further evaluated quantitatively according to the reliability growth test evaluation method. It not only realizes quantitative evaluation based on RET test data, but also derives a variable acceleration model with acceleration factor varying with time. Finally, the feasibility and effectiveness of the application of the method in engineering are preliminarily verified by combining specific examples, which provides a new idea for product reliability evaluation research based on reliability enhancement test data.
References


